Yavorsky - Detlaf: Handbook of Physics

11.4.4. *Internal friction (viscosity)* is associated with the occurrence of friction forces between two layers of the gas or liquid which flow parallel to each other at different speeds. The cause of internal friction, or viscosity, is the transport of momentum by the molecules from one layer to the other. *Newton's equation* for viscosity in the one-dimensional problem, when $v = v(x)$, is

$$dF = -\eta \frac{dv}{dx} dS$$

where dF = force of internal friction acting on an area dS of the surface of the layer

$\frac{dv}{dx}$ = velocity gradient of the motion of the layers in direction x which is perpendicular to the surface of the layer

η = *coefficient of internal friction* (or *of viscosity*) or, for short, the *viscosity*, which is numerically equal to the friction force between two layers of unit area at a unit velocity gradient.

According to elementary kinetic theory

$$\eta = \frac{1}{3} \bar{u} \rho \bar{\lambda}$$

where $\bar{u}$ = average velocity of thermal motion of the molecules

$\bar{\lambda}$ = mean free path

ρ = density of the gas.

In more precise theory, the multiplier $\frac{1}{3}$ is replaced by a factor φ which depends upon the nature of the interaction between the molecules. In collision of molecules regarded as smooth hard spheres, $\varphi = 0.499$. In more precise models of the forces of interaction, φ becomes an increasing function of the temperature.

11.4.5. The thermal conductivity K and viscosity η do not depend upon the density of the gas because the product $\rho\bar{\lambda}$ does not depend upon ρ. The viscosity of gases increases with temperature in proportion to $\sqrt{T}$.

stationary unlimited fluid, and the fluid is equivalent to the problem of the interaction between a motionless body and a steady stream of fluid, flowing around the body, whose velocity $\mathbf{v}_0$ at a point far upstream of the body equals $-\mathbf{u}$.

19.6.2. The Navier-Stokes equation (see 19.3.2) for steady flow of a fluid in the absence of mass forces is of the form

$$(\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho} \operatorname{grad} p + \nu \Delta \mathbf{v} + \left(\frac{\zeta}{\rho} + \frac{\nu}{3}\right) \operatorname{grad} \operatorname{div} \mathbf{v}$$

If the flow of a stream of incompressible fluid ($\operatorname{div} \mathbf{v} = 0$) around the body corresponds to low values of the Reynolds number $\left(\mathbf{Re} = \frac{v_0 l}{\nu} \ll 1\right)$, where l is the characteristic dimension of the body, so that the inertial term $(\mathbf{v}\nabla)\mathbf{v} \ll \nu\Delta\mathbf{v}$, the equation can be written in the following approximate form

$$\eta \Delta \mathbf{v} - \operatorname{grad} p = 0 \quad \text{or} \quad \Delta \operatorname{curl} \mathbf{v} = 0$$

19.6.3. The *force of resistance* **F** of a fluid, acting on a sphere moving slowly in the fluid, is determined by *Stokes' law*:

$$\mathbf{F} = -6\pi\eta R\mathbf{u}$$

where R = radius of the body (sphere)

$\mathbf{u}$ = velocity of the body

η = dynamic viscosity of the fluid.

This law is valid for cases in which $\mathbf{Re} \ll 1$, $\left(\mathbf{Re} = \frac{uR\rho}{\eta}\right.$, where ρ is the density of the fluid$\left.\right)$.

The terminal velocity u of a solid falling in a viscous fluid by the action of the gravity force is equal, within the limits of application of Stokes' law, to

$$u = \frac{2R^2 g(\rho' - \rho)}{9\eta}$$

where ρ' = density of the sphere

g = free fall acceleration.