

Η διαφορά φάσης των δυο ταλαντώσεων που εκτελεί ένα υλικό σημείο μετά την συμβολή των κυμάτων στο σημείο αυτό είναι: ($r_2 > r_1$).

$$\Delta\phi = \frac{2\pi}{T} \cdot \Delta t = \frac{2\pi}{T} \cdot \frac{r_2 - r_1}{v_s} \Rightarrow \Delta\phi = \frac{2\pi}{\lambda} (r_2 - r_1) \quad (1a)$$

$$κ' A' = 2A \cdot \left| \sin\left(\pi \cdot \frac{r_2 - r_1}{\lambda}\right) \right| \xrightarrow{(1a)} A' = 2A \left| \sin\left(\frac{\Delta\phi}{2}\right) \right| \quad (1b)$$

τούμβ. = $t_2 = \frac{r_2}{v_s}$ Τότε $y_2 = 0$ κ' $v_2 = +\omega A$.

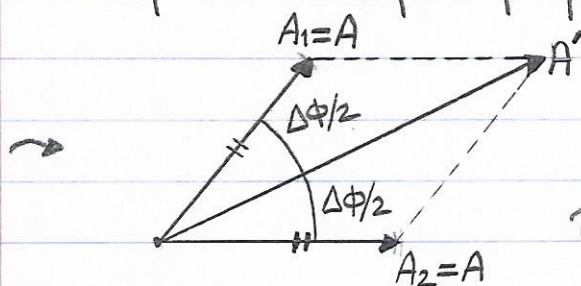
$$κ' y_1 = A \eta\mu\left(\frac{2\pi}{T} \cdot \frac{r_2}{v_s} - \frac{2\pi r_1}{\lambda}\right) \Rightarrow y_1 = A \eta\mu\left[\frac{2\pi}{\lambda} (r_2 - r_1)\right] \xrightarrow{(1a)}$$

$$\Rightarrow y_1 = A \cdot \eta\mu(\Delta\phi) \quad \text{ομοίως } v_1 = \omega A \sigma\upsilon\upsilon(\Delta\phi)$$

Τελικά: τούμβ. έχουμε $y = y_1 = A \eta\mu(\Delta\phi)$. (2a)

$$v = \omega A \cdot (1 + \sigma\upsilon\upsilon(\Delta\phi)) \quad (2b)$$

και με τα περιτρεφόμενα: ($t = \text{τούμβ.}$) \rightsquigarrow



$$y = A' \cdot \eta\mu[\omega(t - \text{τούμβ.}) + \phi_0] \quad (3)$$

$$\phi_0 \in [0, 2\pi) \text{ rad}$$

A. Περίπτωση:

$$\Delta t = \frac{3T}{2} \xrightarrow{(1a)} \Delta \phi = 3\pi \text{ rad} \xrightarrow{(1b)} A' = 0. \Rightarrow \textcircled{\Gamma}$$

B. Περίπτωση:

$$\Delta t = 3T \xrightarrow{(2a)} \Delta \phi = 6\pi \text{ rad} \xrightarrow{(2b)} A' = 2A$$

$$\textcircled{2a} \leadsto y = y_1 = 0, \quad \textcircled{2b} \leadsto v = +2\omega A \uparrow \Rightarrow \textcircled{A}$$

$$\textcircled{3} \leadsto y = 2A \cdot \eta \mu \left[\omega \left(t - \frac{r_2}{v_s} \right) + 0 \right], \quad t \geq \frac{r_2}{v_s}$$

Επομένως:

i. για $\Delta t = k \cdot \frac{T}{2} \leadsto \Delta \phi = k \cdot \pi \leadsto A' = 0, k = \text{περιττός}$

ii. για $\Delta t = k \cdot T \leadsto \Delta \phi = 2k\pi \leadsto A' = 2A, k \in \mathbb{Z}$

Γ. Περίπτωση:

Από την γραφική παράσταση την $t_2 = t_{\text{συμμ.}} \leadsto$

$$\leadsto y = y_1 \quad \text{κ' από την } \textcircled{2b} \quad v > 0 \text{ για } \sin(\Delta \phi) \neq -1$$

άρα σωστή η $\textcircled{\Gamma}$

Παραδείγματα: $y_{\pi_1, \pi_2} = 2 \eta \mu \left(\frac{2\pi}{3} \cdot t \right)$, SI

$$f = \frac{1}{3} \text{ Hz}, v_s = 2 \text{ m/s} \sim \lambda = 6 \text{ m.}$$

1. $r_2 = 5.8 \text{ m} \sim t_{\text{συμβ.}} = 2.9 \text{ sec}$

$$r_1 = 4 \text{ m}$$

$$\textcircled{1a} \rightarrow \Delta\phi = 0.6\pi \text{ rad}, \textcircled{1b} \rightarrow A' = 4 \left| \sin(0.3\pi) \right| = 2.35 \text{ m}$$

τσυμβ.: $\textcircled{2a} \rightarrow y = y_1 = 2 \eta \mu(0.6\pi) = +1.90 \text{ m.}$

$$\textcircled{2b} \rightarrow v = \frac{4\pi}{3} (1 + \sin(0.6\pi)) = +0.92 \cdot \pi \text{ m/s.}$$

Από τα περιστρεφόμενα:

$$y = 2.35 \eta \mu \left[\frac{2\pi}{3} \cdot (t - 2.9) + 0.3\pi \right], t \geq t_{\text{συμβ.}} = 2.9 \text{ sec.}$$

2. $r_2 = 6 \text{ m} \sim t_{\text{συμβ.}} = 3 \text{ sec}$

$$r_1 = 4 \text{ m}$$

$$\textcircled{1a} \rightarrow \Delta\phi = \frac{2\pi}{3} \text{ rad}, \textcircled{1b} \rightarrow A' = 4 \left| \sin\left(\frac{\pi}{3}\right) \right| = 2 \text{ m.}$$

τσυμβ.: $\textcircled{2a} \rightarrow y = y_1 = 2 \eta \mu\left(\frac{2\pi}{3}\right) = +\sqrt{3} \text{ m}$

$$\textcircled{2b} \rightarrow v = \frac{4\pi}{3} \cdot (1 + \sin(\pi/3)) = +2\pi \text{ m/s.}$$

Από τα περιστρεφόμενα:

$$y = 2 \cdot \eta \mu \left[\frac{2\pi}{3} \cdot (t - 3) + \frac{\pi}{3} \right], t \geq t_{\text{συμβ.}} = 3 \text{ sec}$$

3. $r_2 = 8\text{ m} \rightsquigarrow t_{\text{συμβ.}} = 4\text{ sec}$

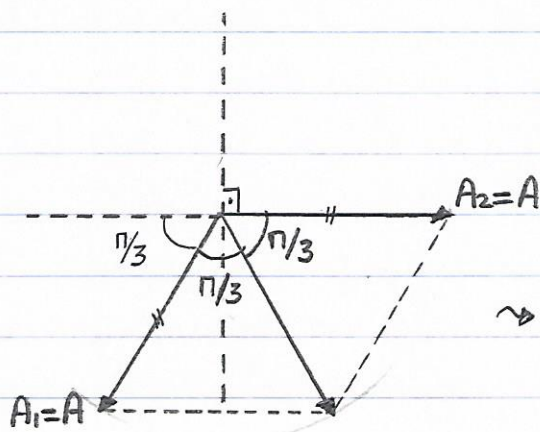
$r_1 = 4\text{ m}$

(1a) $\rightsquigarrow \Delta\phi = \frac{4\pi}{3}\text{ rad}$, (1b) $\rightsquigarrow A' = 4 \left| \sin\left(\frac{2\pi}{3}\right) \right| = 2\text{ m}$.

τσυμβ.: (2a) $\rightsquigarrow y = y_1 = 2 \cdot \eta\mu\left(\frac{4\pi}{3}\right) = -\sqrt{3}\text{ m}$

(2b) $\rightsquigarrow v = \frac{4\pi}{3} \left(1 + \sigma\omega\left(\frac{4\pi}{3}\right)\right) = +\frac{2\pi}{3}\text{ m/s}$

Από τα περιστρεφόμενα:



$\rightsquigarrow y = 2 \cdot \eta\mu\left[\frac{2\pi}{3}(t-4) + \left(\pi + \frac{2\pi}{3}\right)\right]$

για $t \geq t_{\text{συμβ.}} = 4\text{ sec}$

4. $r_2 = 6.4\text{ m} \rightsquigarrow t_{\text{συμβ.}} = 3.2\text{ sec}$

$r_1 = 4\text{ m}$

(1a) $\rightsquigarrow \Delta\phi = 0.8\pi\text{ rad}$, (1b) $\rightsquigarrow A' = 4 \cdot \left| \sin(0.4\pi) \right| = 1.236\text{ m}$

τσυμβ.: (2a) $\rightsquigarrow y = y_1 = 2 \cdot \eta\mu(0.8\pi) = +1.175\text{ m}$

(2b) $\rightsquigarrow v = \frac{4\pi}{3} \cdot (1 + \sigma\omega(0.8\pi)) = +0.255 \cdot \pi\text{ m/s}$.

Από τα περιστρεφόμενα:

$y = 1.236 \cdot \eta\mu\left[\frac{2\pi}{3}(t-3.2) + 0.4\pi\right]$, $t \geq t_{\text{συμβ.}} = 3.2\text{ sec}$

5. $r_2 = 8.4 \text{ m} \rightarrow \text{τούμβ.} = 4.2 \text{ sec}$

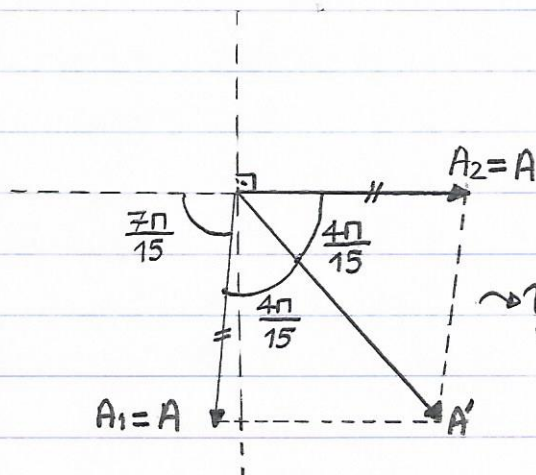
$r_1 = 4 \text{ m}$

(1α) $\rightarrow \Delta\phi = \frac{22\pi}{15} \text{ rad}$, (1β) $\rightarrow A' = 4 \cdot \left| \sin\left(\frac{11\pi}{15}\right) \right| = 2.676 \text{ m}$

τούμβ.: (2α) $\rightarrow y = y_1 = 2 \cdot \eta\mu\left(\frac{22\pi}{15}\right) = -1.989 \text{ m}$

(2β) $\rightarrow v = \frac{4\pi}{3} \cdot \left(1 + \sin\left(\frac{22\pi}{15}\right)\right) = +1.194 \cdot \pi \text{ m/s}$

Από τα περιστρεφόμενα:



$\rightarrow y = 2.676 \cdot \eta\mu\left[\frac{2\pi}{3}(t - 4.2) + \left(\pi + \frac{11\pi}{15}\right)\right]$

για $t \geq \text{τούμβ.} = 4.2 \text{ sec}$.

6. $r_2 = 9.6 \text{ m} \rightarrow \text{τούμβ.} = 4.8 \text{ sec}$

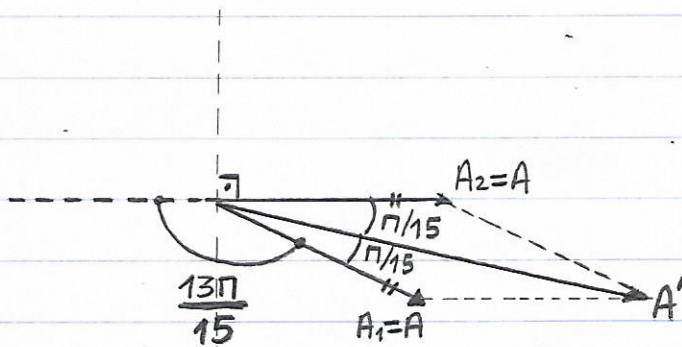
$r_1 = 4 \text{ m}$

(1α) $\rightarrow \Delta\phi = \frac{28\pi}{15} \text{ rad}$, (1β) $\rightarrow A' = 4 \cdot \left| \sin\left(\frac{14\pi}{15}\right) \right| = 3.912 \text{ m}$

τούμβ.: (2α) $\rightarrow y = y_1 = 2 \cdot \eta\mu\left(\frac{28\pi}{15}\right) = -0.813 \text{ m}$

(2β) $\rightarrow v = \frac{4\pi}{3} \cdot \left(1 + \sin\left(\frac{28\pi}{15}\right)\right) = +2.551 \cdot \pi \text{ m/s}$

Από τα περιστρεφόμενα: \rightarrow



$$\rightarrow y = 3.912 \cdot \eta\mu \left[\frac{2\pi}{3} \cdot (t - 4.8) + \left(\pi + \frac{14\pi}{15} \right) \right]$$

για $t \geq t_{\text{συμβ.}} = 4.8 \text{ sec.}$

Παρατήρηση:

Όταν : $0 < \Delta\phi < \pi \text{ rad}$: τότε $\phi_0 = \frac{\Delta\phi}{2}$ (1,2,4)

: $\pi \text{ rad} \leq \Delta\phi < 2\pi \text{ rad}$: τότε $\phi_0 = \pi + \frac{\Delta\phi}{2}$ (3,5,6)